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D3.4: Existence of forbidden frequency bands due to the presence of metamaterials at the geophysical scale

"All waves behave in a similar way" Propagation and Group Velocity, 1960) — Léon Brillouin (Wave

I. Introduction

Wave control efforts in elasticity have been inspired by previous accomplishments in electromagnetism and acoustics through structured composite materials. Waves propagating through such materials behave in ways not necessarily observed in nature and thus gave rise to a new type of wave propagation physics. Lately, this concept of wavefield manipulation on smaller scales have been extended and applied to seismic waves on the geophysical scale. It has been demonstrated that seismic waves can be controlled by either having a spatially periodic distribution of scatterers arranged in the soil or by sub-wavelength structures coupled to the ground which have local resonances resulting in anomalous dispersion of waves. When the seismic 'meta-material' is properly engineered, it is possible to isolate ground vibrations within a specific range of frequencies referred to as *bandgaps*. If the frequency of propagating waves lie within these bandgaps, their amplitude gets attenuated by a mechanism of either local resonance or wave interferences within the periodic system. Owing to the unique properties and potential applications of seismic metamaterials, it has created a great deal of interest within the broad scientific community. Some relevant experimental and numerical studies have been reviewed in the following sections. In this report, we pay attention to implications the current studies can have for an urban-like environment.

II. Review of Seismic Metamaterials

A. Periodic Soil Structures





Materials having density and elastic constants as periodic functions of the position are referred to as periodic structures. Drawing on the ideas of photonic [1, 2] and phononic crystals [3, 4], civil engineers from Ménard company carried out two full-scale field experiments [5, 6] to investigate the concept of periodic structures in the context of seismic surface waves on the decimeter and meter scale. By structuring the top layer of soft soil with a regular arrangement of cylindrical and empty boreholes, the distribution of seismic energy was drastically changed (seen in the right panel of Fig. 1).



FIGURE 1: (Left) Realization of a seismic metamaterial: Full-scale experiment conducted by Ménard Company with empty boreholes in the soil. (Right) The boreholes' effect on the energy field - the difference of the measured energy field with and without the boreholes [5].

Bragg scattering is exploited to obtain bandgaps by constructing such regular structures. The seismic barrier need not necessarily consist of boreholes or a material softer than soil. Seismic wave cancellation can also be achieved by having inhomogeneities which are relatively dense and/or stiff compared to the host material. A small part of the transmitting energy is reflected at these inhomogeneities. However, the wavelength scattered would still depend on the spacing between the scatterers. Using civil engineering construction materials like concrete and steel for structuring soil can be a good way to control the seismic surface waves. Generally, construction of structures in loose and soft strata needs adequate stability of the supporting and surrounding soil. In geotechnical engineering, deep foundations are designed to achieve the required stability and are generally composed of a series of slender piles driven into the ground to a required depth. These piles





structures which are relatively much stiffer than the host soil medium can be considered as cylindrical inclusions arranged in the soil to behave like a seismic barrier. An early theoretical study [9, 10] showed that a row of rigid cylindrical piles has shielding potential based on scattering and diffraction for incident Rayleigh waves.

Various other efforts were made towards developing a seismic barrier design for low-frequency Rayleigh waves in the 5-10 Hz range through finite-element simulations [11-14]. Cylindrical concrete columns embedded in soil (modelled as a thick plate), when arranged in certain configurations [11] can be efficient not only for seismic protection but also cause less disturbance of the wavefield giving good invisibility characteristics. For low-frequency seismic waves which typically are a few ten to hundred meters long, we need to place inclusions of spacing and dimension such that we are within or close to the Bragg regime. Some studies analyzed bandgaps for periodic inclusions of decameter size, which are either hollow or filled with another material [12, 13]. The band structures obtained for both lamb waves (thick plate) and surface waves (half-space) were formed at low frequencies mostly within the range of 1-10 Hz, albeit not appreciably wide in the case of surface waves.

An interesting zero-frequency stop band phenomenon is observed when closely spaced piles of practical dimensions are clamped to the bedrock [14]. Bandgaps with cut-off frequency as high as 30 Hz were obtained by exploiting this clamping effect. However, in a real scenario, having perfect zero Dirichlet boundary conditions may not be feasible, and this solution may only work in specific geological conditions where the stiff layer is reasonably close to the surface. These studies indicate that the building foundation structures in an urban-like scenario - which can be considered to be periodic, scatter seismic waves of the same order as their spacing to give rise to bandgaps. And if the foundation structure can be clamped into an underlying stiffer subsurface layer, the bandgaps can start from ultra-low frequencies.

B. Locally-Resonant Structures

The concept of trapping wave energy using local resonances is an alternative means of mitigating seismic surface waves. In the case of periodic structures, the spacing between the scatterers 'a' is typical of the order of the wavelength ' λ ' ($a \approx \lambda/2$) to be within the Bragg regime. However, the advantage with locally-resonant structures is their deep subwavelength character [15], i.e. the size of a single unit can be much smaller than the wavelength, $a << \lambda$ (at least an order less than the wavelength). To block incoming Rayleigh waves, the individual units of the seismic





metamaterial contain resonating structures coupled to the ground - either attached to the surface or placed in the top layer of the soil such that they are excited by the surface waves. However, the arrangement of these individual units need not necessarily be periodic, since resonances drive the bandgaps. It is the spatial density and the quality factor of the resonators that decide the extent and efficiency of the bandgap. The energy of propagating waves is trapped within many sub-wavelength resonating units, thus causing it to dampen before reaching the structure.



FIGURE 2: (Left) Meta-boxes and (Right) Pressure distribution due to the meta-barrier at 10 Hz [8].

Prior to the field experiments by Ménard, a first of its kind seismic waveguide was proposed, which was based on acoustic metamaterial [7, 8]. The shell-type hollow boxes with holes in their sides, based on Helmholtz resonators, the so-called meta-cylinders and meta-boxes (Fig. 2) can be placed side-by-side to convert seismic energy into sound or heat by the resonance of the air in the cavity. This was successfully demonstrated by finite element numerical simulations. To better understand the local resonance phenomenon shown by surface resonators (termed as a 'metasurface') and to investigate the control of Lamb waves, a series of lab experiments [16-20] involving a set of aluminium rods attached to an aluminium plate (as shown in Fig. 3), were carried out. Strong hybridization between the A_0 plate mode with the compressional resonances of the rods opens wide bandgaps [16]. However, the flexural vibrations of rods are weakly excited by the A_0 mode. To separate the Bragg effect from that of local resonance, the rods were also arranged in a spatially disordered way. Energy trapping by introducing a defect inside the metamaterial can have meaningful inferences for geophysical scale applications where buildings act as vertical beams attached to the ground surface [17]. Such a study where one can predict which buildings/regions in the city will be more prone to shaking can also help in urban seismic hazard assessment. A follow-





up of the previous experiments was to study the effect of the configuration/layout of the rods [18]. Specifically, one configuration where the rods were concentrically arranged within a circular region with their heights steadily increasing towards the centre (Fig. 3) showed that back-scattering could be prevented and flexural waves can have wide stop-bands.



FIGURE 3: (Left) Setup for the lab-scale experiment with plate and rods and dispersion curves [16, 20]. (Right) Plate with a cluster of rods arranged such that the central region is cloaked [18].

To verify the results of the plate with resonators on the geophysical scale, a field experiment was conducted in a forest with closely spaced trees [21]. In this initial experiment, just two seismometers were deployed (one inside the forest and the other outside) to measure the ambient field. In the broadband transmission of energy, an apparent fall is seen near the longitudinal resonance frequencies of the trees acting as a 'natural metamaterial'. The 2D spectral element simulations not only confirm this but also reveal the hybridization of the Rayleigh waves due to the interaction with the trees in the bandgap frequency range. Two important observations were made: first, the hybridization phenomenon in a half-space is the surface wave diversion into the bulk (Rayleigh waves converted to shear waves), whereas there is a total reflection in the incidence direction for a plate. Secondly, in the half-space, the bandgap width is determined by the resonance and anti-resonance points. As an extension of this study with a forest of trees, the concept of a metawedge was also explored [22]. The two contracting phenomena were observed on introducing





a gradient in the tree height: a classical metawedge when rod height is increasing along the wavefield direction and an inverse metawedge when the heights are gradually decreasing along the direction of incidence (see right panel of Fig. 4). This results in rainbow trapping effect and conversion of Rayleigh waves to shear waves, respectively. This study also has meaningful implications for seismic waves propagating within an urban area. A layout of buildings of increasing height away from the centre of the city (inverse wedge) can filter surface waves by diverting them into the bulk, thus protecting the more critical downtown area. An experimental study with waves of ultrasonic regime [23] confirmed this concept of Rainbow trapping and mode conversion of Rayleigh waves.



FIGURE 4: (Left) Top - 2D spectral element simulation with trees over a half-space, Bottom – Field experiment layout and spectral ratio plot [21]. (Right) 2D spectral element simulations for classic and inverse metawedge where two contrasting wave phenomena are observed [22].

Another experiment in a pine tree forest near Mimizan, France, with a very dense array of seismic sensors helped in further understanding the dispersion inside the forest metamaterial with higher accuracy [25]. The frequency bandgaps and dispersion curves were validated by means of the geophysical measurements as well as numerical simulations (Fig. 5). A more recent study [28]





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investigated in more depth the surface wave dynamics in the METAFORET experiment. The manner in which surface wave modes couple with subwavelength trees leading to change in their polarization (ellipticity) at the tree resonance frequencies was studied. This analysis found that there is a leakage in the higher-order surface mode having a different ellipticity which is caused by the trees. The horizontal component of the Rayleigh waves does not couple efficiently with the flexural resonances of the trees, therefore creating narrow bandgaps compared to those formed by longitudinal tree resonances. The coupling of the trees with the dipolar excitation from the incident waves in the horizontal direction may not be very strong, although there should be some shear waves excited by the trees acting as secondary sources. However, in the plate and rods experiment, the reason for negligible flexural bandgaps is relatively simple to understand because the elongated rods had very weak flexural resonance from the excited A₀ lamb mode. Also, the contrast in rigidities between the plate and resonator, cause the flexural deformations of the resonators to couple weakly to the plate [24]. Thus with increasing flexibility of the plate, the flexural resonances of rods can become more important. The flexural resonances being predominant in building structures at low frequencies, this could be a subject of detailed analysis in an urban-like environment.



FIGURE 5: Numerical Simulations versus real data for the METAFORET experiment [25]





Since surface waves travel in the superficial layers of the ground, the resonating elements can also be placed in the top layers of the soil, and when excited by the surface wave, it can exhibit multiple resonant modes. Previous studies which studied this kind of metasurface [28-40] considered placing a heavy mass suspended by elastic bearings (Fig. 6) or mass-in-mass resonators. The bandgaps are opened near the resonance frequencies, which allows for tuning the bandgaps to the required target frequency to be attenuated, which can be even below 5 Hz. However, the drawback with these designs was that the bandgaps are only produced in a narrow band close to the resonance frequencies and are not wide enough to be considered as efficient seismic barriers. Another way is to have a graded-mass array with a 3D periodic arrangement (cubic array) of many such resonator units [36, 37] instead of placing them only in the surface layer. This helps in shielding some of the body waves from reaching the foundation of the structure.



FIGURE 6: (Left) Schematic of an engineered metabarrier as a shield from seismic surface waves (Right) Dispersion curves showing the resonances and the Rayleigh wave BG [31]

III. Dense Urban areas as metamaterials for seismic waves

The problem of site-city interaction and soil-structure coupling has been visited in the past [41, 42]. It is well known that seismic waves propagating in a basin of soft sediments can strongly interact with the buildings due to the frequency of the waves in the basin matching the resonance frequencies of the buildings. The 1985 earthquake that affected Mexico City is a textbook example where the site conditions were responsible for the amplified long-period ground motion. The building resonance in the core of the city situated on a lake bed zone resulted in a long period, monochromatic beating in ground motion [41]. The combined effects of coupling of the soil-structure-





foundation system and the site-city interaction are of genuine interest from the perspective of metamaterial physics.

As a natural extension of the forest experiment; tall buildings clustered together (a scaled-up version of trees) could excitingly act as strong sub-wavelength resonators when they interact with seismic waves. These massive structures having high mass and contrast in elastic properties which are also well coupled to the ground could allow efficient capture of seismic energy. The buildings act as secondary sources and affect the ground motion within the metamaterial. They can potentially scatter, deviate and even cancel the ground vibration. Moreover, unlike the case of trees, we expect to observe the hybridization of surface waves at much lower frequencies which are of primary interest in earthquake engineering. Also, the results from the various lab and field experiments of the METAFORET project can undoubtedly have many urban-scale implications. Similarly, wind farms with massive turbines can also act as a scaled version of trees.

Some preliminary numerical studies [43-46] have attempted to study the interactions between the buildings, and between buildings and seismic waves in the context of metamaterials. A recent field experiment in the city of Quito, Ecuador, in a semi-urban area [46] was conducted to verify the concept of an urban metamaterial by deploying instrumentation in the field and the buildings. The bottom line from these preliminary studies is that interactions between seismic wavefield and buildings exist. The goal is to discern the nature of seismic waves propagating in a dense urban environment which can act as a 'geophysical metamaterial' on the scale of a city and eventually help design the future metacities. This idea can potentially be transformative to the field of urban seismology. Our understanding, however, is still rudimentary, and further experimental studies are required for a better understanding of this complex physics.

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